Published by Institute of Physics Publishing for SISSA/ISAS
Received: *August 6, 2002*
Accepted: *October 21, 2002*

The non-abelian D-brane effective action through order α'^4

Paul Koerber* **and Alexander Sevrin**

Theoretische Natuurkunde, Vrije Universiteit Brussel
Pleinlaan 2, B-1050 Brussels, Belgium
E-mail: koerber@tena4.vub.ac.be, asevrin@tena4.vub.ac.be

Abstract: Requiring the existence of certain BPS solutions to the equations of motion, we determine the bosonic part of the non-abelian D-brane effective action through order α'^4. We also propose an economic organizational principle for the effective action.

Keywords: D-branes, Brane Dynamics in Gauge Theories.

*Aspirant FWO.

Contents

1. Introduction

The bosonic worldvolume degrees of freedom of a single Dp-brane are $9-p$ scalar fields and a U(1) gauge field in $p+1$ dimensions [1]. In the limit of slowly varying fields, the effective action which determines the dynamics of the D-brane at low energies, is known to all orders in α'. It is the ten-dimensional supersymmetric Born-Infeld action, dimensionally reduced to $p+1$ dimensions [2]. Its supersymmetric extension was obtained in [3]. The knowledge of the full effective action was crucial for numerous applications.

Once several, say n, D-branes coincide, the gauge group is enhanced from U(1) to U(n), [4]. The full non-abelian extension of the Born-Infeld theory is not known yet. This is mainly due to two (related) reasons. As all fields take their values in the adjoint representation of U(n), an ordering prescription is needed. Besides this we also need to include derivative terms. Indeed the limit of (covariantly) constant fields is intrinsically related to the abelian limit as $D_c F_{ab} = 0$ implies that $[F_{dc}, F_{ab}] = 0$. This can be seen in another way. In the abelian case, the effective action consists of terms of the form (in a very schematic notation) $g^{-2}\alpha'^{2m-2}\partial^{2n}F^{2m-n}$. Performing the following rescaling,[1] $x \to \varepsilon^{-1}\beta^{-1}x$, $A \to \beta A$, $\alpha' \to \varepsilon^{-1}\beta^{-2}\alpha'$, $g \to \varepsilon\beta^2 g$, we find that $g^{-2}\alpha'^{2m-2}\partial^{2n}F^{2m-n} \to \varepsilon^n g^{-2}\alpha'^{2m-2}\partial^{2n}F^{2m-n}$. Sending $\varepsilon \to 0$, the derivative terms vanish and the Born-Infeld action remains. In the non-abelian case we still have that $g^{-2}\alpha'^{2m-2}D^{2n}F^{2m-n} \to \varepsilon^n g^{-2}\alpha'^{2m-2}D^{2n}F^{2m-n}$. However we also have that $D\cdot = \partial\cdot + [A,\cdot] \to \varepsilon\beta(\partial\cdot + \varepsilon^{-1}[A,\cdot])$ and $F = \partial A + [A,A] \to \varepsilon\beta^2(\partial A + \varepsilon^{-1}[A,A])$ which makes the $\varepsilon \to 0$ limit meaningless. Other arguments for the relevance of the derivative terms in both the abelian and non-abelian case were given in [5].

At this moment the non-abelian effective action is known through $\mathcal{O}(\alpha'^3)$ including the terms quadratic in the gauginos. The leading order term of the effective action for n coinciding Dp-branes is the ten-dimensional $N = 1$ supersymmetric U(n) Yang-Mills

[1] The parameter β reflects the only scaling freedom left. It can e.g. conveniently be chosen such that g and α' remain fixed in the limit $\varepsilon \to 0$.

theory dimensionally reduced to $p+1$ dimensions. There are no $\mathcal{O}(\alpha')$ corrections. The bosonic $\mathcal{O}(\alpha'^2)$ were first obtained in [6] and [7] while the fermionic terms were obtained in [8] and [9]. In [8] supersymmetry fixed the correction while in [9] a direct calculation starting from four-point open superstring amplitudes was used. At higher orders a direct calculation becomes problematic[2] and one has to rely on indirect methods. A very powerful strategy was proposed in [10].[3]

Central in the approach of [10], was the existence of higher-dimensional generalizations of instantons: stable holomorphic bundles. They solve the Yang-Mills equations of motion and in a D-brane context they correspond to BPS configurations. The effective action can be viewed as a deformation of the Yang-Mills action. Requiring that stable holomorphic bundles, or some deformation thereof, solve the equations of motion yields a recursive method to construct the effective action. While this approach becomes very tedious at higher orders, it has the great advantage that its algorithmic nature allows for a computerized approach. To this end a program in Java, an object oriented language based on the syntax of C, was developed [12]. In [13] the method was successfully applied to determine the bosonic $\mathcal{O}(\alpha'^3)$ terms in the effective action. Very recently, in [14], supersymmetry was used not only to confirm the results of [13] but to construct the terms quadratic in the gauginos through this order as well. The full effective action through this order was tested in [15] and there is no doubt left that the result is indeed correct.

In the present paper we extend the results of [13] and we obtain the bosonic terms in the effective action at order α'^4. Furthermore we will propose a very economic way to organize the terms in the effective action.

Throughout the paper we will put $2\pi\alpha' = 1$.

2. General strategy

We consider a U(n) Yang-Mills theory in $2p$ dimensions with a euclidean signature. Its equations of motion are given by,[4]

$$D_a F_{ab} = 0\,. \tag{2.1}$$

Switching to complex coordinates $z^\alpha = (x^{2\alpha-1} + i x^{2\alpha})/\sqrt{2}$, $\bar{z}^{\bar{\alpha}} = (x^{2\alpha-1} - i x^{2\alpha})/\sqrt{2}$, eq. (2.1) reads,

$$\begin{aligned} 0 &= D_{\bar{\alpha}} F_{\alpha\bar{\beta}} + D_\alpha F_{\bar{\alpha}\bar{\beta}} \\ &= D_{\bar{\beta}} F_{\alpha\bar{\alpha}} + 2 D_\alpha F_{\bar{\alpha}\bar{\beta}}\,, \end{aligned} \tag{2.2}$$

where we used the Bianchi identities. One sees that imposing the following linear relations between the field-strengths,

$$F_{\alpha\beta} = F_{\bar{\alpha}\bar{\beta}} = 0\,, \tag{2.3}$$

and

$$\sum_\alpha F_{\alpha\bar{\alpha}} \equiv F_{\alpha\bar{\alpha}} = 0\,, \tag{2.4}$$

[2]We thank Stefan Stieberger for explaining this to us.

[3]A very different approach was recently proposed in [11].

[4]Throughout this paper, we write all indices down and we sum over repeated (real and complex) indices.

JHEP10(2002)046

provides a solution to the equations of motion. In four dimensions, these are exactly the standard instanton equations. In general, eq. (2.3) defines a holomorphic bundle and eq. (2.4) requires it to be stable [16]. These solutions were discovered in [17].

Following [13], we subsequently construct, order by order in α', all possible terms of the correct dimension which can be build out the field-strength and its derivatives. More concretely, at order α'^m, we write down all possible terms having $2n$ covariant derivatives, with $n \in \{0, 1, \ldots, m\}$, and $m - n + 2$ field-strength tensors, taking the cyclicity of the group trace into account. Each of these terms gets an arbitrary coupling constant. The same game has to be played for the most general deformation of eq. (2.4), which through this order has the form

$$F_{\alpha\bar{\alpha}} = \sum_{n=1}^{m} \mathcal{F}_{(n)}\,, \tag{2.5}$$

where $\mathcal{F}_{(n)}$ with $n < m$ were already determined at lower orders and $\mathcal{F}_{(m)}$ is the most general polynomial of terms having $2n$ derivatives and $m - n + 1$ field-strengths, where $n \in \{0, 1 \ldots, m\}$. Here again we leave the coupling constants free. Surprisingly, the coupling constants in *both* the lagrangian and the stability condition are determined by requiring that eqs. (2.3) and (2.5) solve the equations of motion. The presence of derivative terms however, complicates the analysis considerably. Indeed when writing down the most general lagrangian at a certain order in α', the following points have to be taken into account,

1. Terms can be related through partial integration.

2. Terms can be related by Bianchi identities.

3. The $[D, D]\cdot = [F, \cdot]$ identities[5] relate certain terms.

4. Certain terms can be eliminated by field redefinitions.

Similar considerations hold, apart from the first point, for the stability condition. We discard the fourth point until we perform the final analysis of the resulting action. The reason for this is that certain field redefinitions affect the complex structure such that eq. (2.3) gets modified and, as a consequence, our method is not entirely field redefinition independent.

Because of the identities of points one, two and three, many terms are dependent. Handling this essentially boils down to choosing an appropriate basis of independent terms in the lagrangian as well as in the stability condition. The basis terms in the lagrangian must be such that there is a neat separation between the irrelevant field redefinition terms and the relevant non field redefinition terms. In concreto, when expressing the result in a certain basis, the coefficients of some of the basis terms can be shifted by field redefinitions where, again, great care is needed because the field redefinitions are also related by the

[5] In their most general form they read: $[D_1 D_2 \ldots D_{n-2} F_{n-1,n}, D_{n+1} \ldots D_{n+m-2} F_{n+m-1,n+m}] = [D_1, [D_2, \ldots [D_{n-2}, [D_{n-1}, D_n]] \ldots]] D_{n+1} D_{n+m-2} F_{n+m-1,n+m}$. But we will use the above shorthand in the rest of the paper.

JHEP10(2002)046

identities of point two and three. The shifts are independent when the number of such field redefinition terms is minimal. In that case, we can bring their coefficients to zero. So only the projection of the result on the non field redefinition terms is relevant.

Of course, there is still a lot of freedom in choosing a basis, so we need some kind of organizational principle to get rid of at least part of this freedom in an economic way.

3. Organizational principle

As in the abelian case, it is possible to classify terms at a certain order in α' according to the number of derivatives. For this, we must get rid of the $[D,D]\cdot = [F,\cdot]$ identities. Consider any linear combination of terms. Now, start with the terms without derivatives and fully symmetrize in the fieldstrength tensors. In this process, we use the $[D,D]\cdot = [F,\cdot]$ identities to convert the introduced commutators of fieldstrengths to commutators of derivatives.[6] Next, we turn to the terms with two derivatives an again fully symmetrize them, whereby a term of the form DF or D^2F is considered as a single entity. Again, terms with more derivatives are added as compensation. We proceed in this fashion order by order in the number of derivatives. Since the resulting terms are symmetrized in the fieldstrength tensors, all "non-abelianality" sits in the covariant derivatives. From now on all terms, whether they be part of the lagrangian, stability condition, field redefinitions or equations of motion, should be thought of as symmetrized. Note that this approach follows the spirit of [18] and we could call it a "generalized symmetrized trace prescription".

A major advantage is that the non-abelian calculation follows the abelian one more closely, since symmetrized terms in the lagrangian lead to symmetrized terms in the equations of motion. Except for the obvious fact that the derivatives are non-commuting there are however some other differences:

1. There are new identities, because in symmetrizing we only used up part of the $[D,D]\cdot = [F,\cdot]$ identities. An example will clarify this. Only one of the identities[7]

$$\begin{aligned} [F_{12}, F_{34}] &= [D_1, D_2]F_{34} \\ [F_{34}, F_{12}] &= [D_3, D_4]F_{12}\,, \end{aligned} \tag{3.1}$$

is used to commute F_{12} and F_{34}. The other one is left in the form:

$$[D_1, D_2]F_{34} + [D_3, D_4]F_{12} = 0\,. \tag{3.2}$$

[6]This means that we push the $[D,D]\cdot = [F,\cdot]$ identities to the right. The reader could wonder why we do not push those identities to the left and use for instance symmetrized derivatives. This leads indeed to fewer terms and fewer identities, because there are no antisymmetrized Bianchi identities. On the other hand the remaining partial integration and Bianchi identities are more complicated and the final result is awful because we get no clear separation between field redefinition and non field redefinition terms

[7]When writing down long equations or results in the rest of the paper, we will use a shorthand notation for the indices i.e. $1, 2, 3, \ldots$ instead of $a_1, a_2, a_3, \ldots$ Repeated indices are still summed over.

JHEP10(2002)046

These kind of identities are related to antisymmetry, as in this case, or to Jacobi identities of fieldstrength commutators. In general they read:

$$
\begin{aligned}
&[D_1,[D_2,\dots,[D_{m-2},[D_{m-1},D_m]]\dots]]D_{m+1}D_{m+2}\dots D_{m+n-2}F_{m+n-1,m+n}+\\
&+[D_{m+1},[D_{m+2},\dots,[D_{m+n-2},[D_{m+n-1},D_{m+n}]]\dots]]D_1D_2\dots D_{m-2}F_{m-1,m}=0\,,
\end{aligned}
\tag{3.3}
$$

and we call them *antisymmetrized Bianchi identities.*

2. Consider the stability condition[8] eq. (2.5):

$$
F_{\alpha\bar{\alpha}}-\mathcal{F}_{(2)}-\mathcal{F}_{(3)}-\cdots=0\,. \tag{3.4}
$$

Somewhere in the equations of motions we will find $\mathrm{Sym}\{F_{\alpha\bar{\alpha}}T\}$, where Sym means "symmetrized in the fieldstrength tensors" and T contains, say, n fieldstrengths. For this to be zero, and thus for the stability condition to solve this piece of the equations of motion, the term $-\mathrm{Sym}\{(\mathcal{F}_{(2)})T\}$ must also be present in the equations of motion at higher order. Here the inner brackets mean that the factors of $\mathcal{F}_{(2)}$ stay together, so we must further symmetrize this term by mixing those factors among the factors of T. Unlike in the abelian case, terms with more derivatives are thus introduced. Interestingly, the number of extra derivatives in these terms must be a multiple of four. Indeed, both $\mathrm{Sym}\{(\mathcal{F}_{(2)})T\}$ and $\mathrm{Sym}\{\mathcal{F}_{(2)}T\}$ are symmetric under the reversion of all factors, so their difference is symmetric as well and hence must contain an *even* number of commutators.

This is the only way in which terms without derivatives — and using also only contributions without derivatives to the stability condition — communicate with terms with derivatives and thus the mechanism that prevents the "ordinary" symmetrized trace to have BPS solutions.

After we found the result in this way, we used a second organizing principle to simplify even further. We tried to use basis terms with as many "groups" of nested covariant derivative commutators as possible. Each group corresponds, using a $[D,D]\cdot=[F,\cdot]$ identity to a commutator of fieldstrengths or equivalently to an algebra structure constant which can be put in front. To clarify this correspondence we write result (4.4) in different ways using $[D,D]\cdot=[F,\cdot]$ identities:

$$
\begin{aligned}
[D_3,D_2]D_4F_{51}D_5[D_4,D_3]F_{12}&=\\
[F_{32},D_4F_{51}]D_5[F_{43},F_{12}]&=\\
[D_4,[D_5,D_1]]F_{32}D_5[D_1,D_2]F_{43}&\,.
\end{aligned}
\tag{3.5}
$$

The first and the third line show that the number of *groups* of commutators of derivatives is important rather than the number of commutators. Although we pushed the $[D,D]\cdot=[F,\cdot]$ identities to the right initially, in this way we can easily see which terms can be pushed

[8]The order α' correction is zero.

JHEP10(2002)046

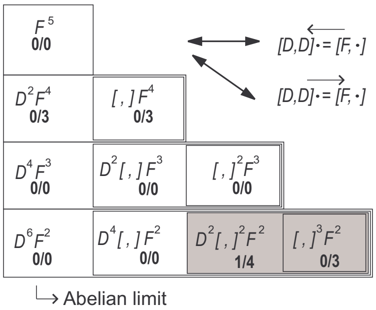

Figure 1: Basis terms at order α'^3. The horizontal classification is by the number of derivatives if the $[D, D]\cdot = [F, \cdot]$ identities are pushed to the left as in the text. The vertical classification is by the number of commutator groups of Ds. Note that when pushing the $[D, D]\cdot = [F, \cdot]$ identities to the right, you can find the terms with the same number of derivatives on the diagonals as indicated. The numbers indicate (#non-zero coefficients in result)/(#all non field redefinition basis terms). The grey area indicates the basis terms potentially used for the result. We tried to simplify as much as possible and put as many as possible coefficients to zero.

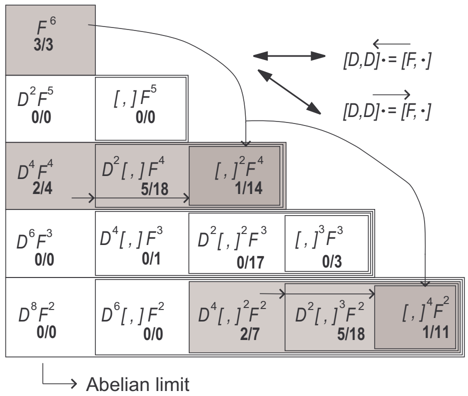

Figure 2: Basis terms at order α'^4. See also the caption of figure 1. The arrows indicate how the abelian result transforms in the non-abelian result. Partial integration and Bianchi identities mix terms with different numbers of commutator groups. Hence the vertical arrows. For the diagonal arrows, see the argument of section 3, point 2. Using our method in the abelian case, there are two groups of terms, the Born-Infeld part and the $\partial^4 F^4$ part. The latter has an arbitrary constant. Note also the new group of terms with 8 derivatives. The dark grey areas are where different groups meet in the full non-abelian case. As a result the arbitrary constant is fixed.

how far in the other direction if necessary. This might be useful when comparing to results following from string amplitude calculations, as they tend to give results where commutators of derivatives are pushed into antisymmetric combinations of fieldstrength tensors.

See figure 1 and 2 for the classification of the terms. Obviously, there is still basis freedom left, so that some terms in the result can move to the right. E.g. result (4.4) can also be written as a sum of terms with 2 and 3 structure constants,[9] however not with terms with only 3 structure constants. Hence the grey area in figure 1.

4. The result

The purely bosonic part of the non-abelian effective action through $\mathcal{O}(\alpha'^4)$ is of the form,

$$\mathcal{L} = \frac{1}{g^2} \left(\mathcal{L}_0 + \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 \right), \tag{4.1}$$

[9] See the result in [14].

where the leading term is simply[10]

$$\mathcal{L}_0 == -\operatorname{Tr}\left\{\frac{1}{4}F^2\right\}. \tag{4.2}$$

Subsequently we have

$$\mathcal{L}_2 = \operatorname{STr}\left\{\frac{1}{8}F^4 - \frac{1}{32}F^2F^2\right\}, \tag{4.3}$$

where STr denotes the symmetrized trace prescription. At this point both the overall multiplicative factor in front of the action as well as the scale of the gauge fields got fixed [13]. The next term is[11]

$$\mathcal{L}_3 = \frac{\zeta(3)}{2\pi^3}\operatorname{Tr}\{[D_3, D_2]D_4F_{51}\, D_5[D_4, D_3]F_{12}\}. \tag{4.4}$$

In fact our method did not fix the overall multiplicative factor in front of this term. As explained in [13] and [14], this was most fortunate as one expects from string theory that the coupling constants of terms which are of odd order in α' are irrational while our method yields, at this order, only rational coupling constants. We determined the coupling constants by comparing the relevant terms to the partial results in [5]. Note that eq. (4.4) looks very different from the expression given in [13]. The reason for this is clear. In [13], we used precisely the opposite organizational strategy as the one proposed in section 3. Comparing eq. (4.4) to the corresponding expression in [13] (or [14] where yet another basis was chosen) shows convincingly that the scheme proposed in section 3 is the most economical.

We now turn to the new result of this paper. The fourth order contribution to the effective action reads (see figure 2)

$$\mathcal{L}_4 = \mathcal{L}_{4,0} + \mathcal{L}_{4,2} + \mathcal{L}_{4,4}\,, \tag{4.5}$$

with

$$\begin{aligned}
\mathcal{L}_{4,0} &= \operatorname{STr}\Bigg(\frac{1}{12}F_{12}F_{23}F_{34}F_{45}F_{56}F_{61} - \frac{1}{32}F_{12}F_{23}F_{34}F_{41}F_{56}F_{65} + \\
&\qquad + \frac{1}{384}F_{12}F_{21}F_{34}F_{43}F_{56}F_{65}\Bigg), \\
\mathcal{L}_{4,2} &= \frac{1}{48}\operatorname{STr}\Bigg(-2\,F_{12}D_1D_6D_5F_{23}D_6F_{34}F_{45} - F_{12}D_5D_6F_{23}D_6D_1F_{34}F_{45} + \\
&\qquad + 2F_{12}\,[D_6, D_1]\,D_5F_{23}F_{34}D_4F_{56} + 3D_4D_5F_{12}F_{23}\,[D_6, D_1]\,F_{34}F_{56} + \\
&\qquad + 2D_6\,[D_4, D_5]\,F_{12}F_{23}D_1F_{34}F_{56} + 2D_6D_5F_{12}\,[D_6, D_1]\,F_{23}F_{34}F_{45} + \\
&\qquad + 2\,[D_6, D_1]\,D_3D_4F_{12}F_{23}F_{45}F_{56} + \\
&\qquad + [D_6, D_4]\,F_{12}F_{23}\,[D_3, D_1]\,F_{45}F_{56}\Bigg),
\end{aligned} \tag{4.6}$$

[10] We use the following notation: $F^m \equiv F_{a_1a_2}F_{a_2a_3}\cdots F_{a_ma_1} \equiv F_{12}F_{23}\cdots F_{m1}$.

[11] All results are of course modulo field redefinition terms.

JHEP10(2002)046

$$
\begin{aligned}
\mathcal{L}_{4,4} = \frac{1}{1440}\mathrm{STr}\Big(& D_6[D_4,D_2]D_5D_5[D_1,D_3]D_6F_{12}F_{34} + \\
& + 4D_2D_6[D_4,D_1][D_5,[D_6,D_3]]D_5F_{12}F_{34} + \\
& + 2D_2[D_6,D_4][D_6,D_1]D_5[D_5,D_3]F_{12}F_{34} + \\
& + 6D_2[D_6,D_4]D_5[D_6,D_1][D_5,D_3]F_{12}F_{34} + \\
& + 4D_6D_5[D_6,D_4][D_5,D_1][D_4,D_3]F_{12}F_{23} + \\
& + 4D_6D_5[D_4,D_2][D_6,D_1][D_5,D_3]F_{12}F_{34} + \\
& + 4D_6[D_5,D_4][D_3,D_2][D_5,[D_6,D_1]]F_{12}F_{34} + \\
& + 2[D_6,D_1][D_2,D_6][D_5,D_4][D_5,D_3]F_{12}F_{34}\Big) .
\end{aligned}
\tag{4.7}
$$

The terms with zero derivatives, $\mathcal{L}_{4,0}$, form of course the symmetrized trace Born-Infeld action, while the abelian limit of the terms with four derivatives, the first two terms of $\mathcal{L}_{4,2}$, can be shown to agree with [19].[12] If we use our method in the abelian case at order α'^4, these terms have an arbitrary constant, since terms with a different number of derivatives have no way of communicating in this case. In the non-abelian case however, the coefficient is fixed at precisely the right value !

Note that there are no terms with two derivatives nor with six derivatives. If there were, they would have had an arbitrary constant since, following the reasoning of point 2 in section 3, the symmetrized trace does not communicate directly with terms in which the number of derivatives is not a multiple of four.

5. Conclusions

In the present paper we calculated the bosonic α'^4 contribution to the non-abelian effective D-brane action. As was already obvious from the $\mathcal{O}(\alpha'^3)$ calculation, [13], [14], the inclusion of derivative terms is unavoidable. The "generalized symmetrized trace prescription" proposed in section 3 gives rise to a very economical way of organizing the action which has the additional advantage that it closely mimics the abelian case.

When determining the coupling constants in the effective action and the deformed stability condition, we had to solve 1816 equations for 546 unknowns. The fact that we found a unique (modulo field redefinitions) solution consists in itself a strong check on our calculation. An independent check would follow the lines proposed in [21], where the mass spectrum in the presence of constant magnetic background fields was calculated from the effective action and compared to the string theoretic result. A careful analysis in [22] showed that the symmetrized trace prescription without derivative terms deviated from the string theoretical results, starting at order α'^4. The derivative corrections which were obtained in this paper should cure this problem. We postpone this check to a future paper. In this context we should also mention the analysis in [23] where the mass spectrum in the presence of constant magnetic background fields combined with the requirement that

[12] Although it is a little involved, this can still be shown by hand. See e.g. of [20, appendix B].

the abelian limit was correctly reproduced, were used to determine the effective action as far as possible. Besides the fact that this method yielded a multi-parameter solution an important ansatz was made: only terms without derivatives were taken into account. Our present paper clearly indicates that this ansatz was too strong.

The present result opens the way to all-order statements. E.g., we already noticed, within our generalized symmetrized trace prescription, that terms without derivatives only communicate with terms with $4, 8, \ldots$ derivatives. It follows that terms with $2, 6, 10, \ldots$ derivatives have arbitrary constants in our method. It might very well turn out that the constants for the terms with 2 derivatives vanish and that these terms are zero, just as in the abelian case [19]. As a consequence, terms with $2+4n$, with $n \in \mathbb{N}$, could vanish as well for the even orders of α'. The all-order lessons we can draw from our method is presently under investigation and we will come back to this in a separate publication. Related to this, a careful analysis of the deformed stability condition has been started.

Finally, we hope that our paper can shed some light on the remarkable results recently obtained in [24].

Acknowledgments

The authors are supported in part by the "FWO-Vlaanderen" through project G.0034.02, in part by the Federal Office for Scientific, Technical and Cultural Affairs through the Interuniversity Attraction Pole P5/27 and in part by the European Commission RTN programme HPRN-CT-2000-00131, in which the authors are associated to the University of Leuven.

References

[1] J. Polchinski, *Dirichlet-branes and Ramond-Ramond charges*, *Phys. Rev. Lett.* **75** (1995) 4724 [hep-th/9510017];
J. Dai, R. Leigh and J. Polchinski, *New connections between string theories*, *Mod. Phys. Lett.* **A 4** (1989) 2073.

[2] E.S. Fradkin and A.A. Tseytlin, *Nonlinear electrodynamics from quantized strings*, *Phys. Lett.* **B 163** (1985) 123;
A. Abouelsaood, C.G. Callan, C.R. Nappi and S.A. Yost, *Open strings in background gauge fields*, *Nucl. Phys.* **B 280** (1987) 599;
R.G. Leigh, *Dirac-Born-Infeld action from Dirichlet sigma model*, *Mod. Phys. Lett.* **A 4** (1989) 2767.

[3] M. Cederwall, A. von Gussich, B.E.W. Nilsson and A. Westerberg, *The Dirichlet super-three-brane in ten-dimensional type-IIB supergravity*, *Nucl. Phys.* **B 490** (1997) 163 [hep-th/9610148];
M. Aganagic, C. Popescu and J.H. Schwarz, *D-brane actions with local kappa symmetry*, *Phys. Lett.* **B 393** (1997) 311 [hep-th/9610249];
M. Aganagic, C. Popescu and J.H. Schwarz, *Gauge-invariant and gauge-fixed D-brane actions*, *Nucl. Phys.* **B 495** (1997) 99 [hep-th/9612080];

M. Cederwall, A. von Gussich, B.E.W. Nilsson, P. Sundell and A. Westerberg, *The Dirichlet super-p-branes in ten-dimensional type-IIA and IIB supergravity*, *Nucl. Phys.* **B 490** (1997) 179 [hep-th/9611159];
E. Bergshoeff and P.K. Townsend, *Super D-branes*, *Nucl. Phys.* **B 490** (1997) 145 [hep-th/9611173].

[4] E. Witten, *Bound states of strings and p-branes*, *Nucl. Phys.* **B 460** (1996) 335 [hep-th/9510135].

[5] A. Bilal, *Higher-derivative corrections to the non-abelian Born-Infeld action*, *Nucl. Phys.* **B 618** (2001) 21 [hep-th/0106062].

[6] D.J. Gross and E. Witten, *Superstring modifications of Einstein's equations*, *Nucl. Phys.* **B 277** (1986) 1.

[7] A.A. Tseytlin, *Vector field effective action in the open superstring theory*, *Nucl. Phys.* **B 276** (1986) 391, erratum *ibid.* **B291** (1987) 876.

[8] E. Bergshoeff, M. Rakowski and E. Sezgin, *Higher derivative super Yang-Mills theories*, *Phys. Lett.* **B 185** (1987) 371;
M. Cederwall, B.E.W. Nilsson and D. Tsimpis, *The structure of maximally supersymmetric Yang-Mills theory: constraining higher-order corrections*, *J. High Energy Phys.* **06** (2001) 034 [hep-th/0102009]; $D = 10$ *super-Yang-Mills at* $O(\alpha^2)$, *J. High Energy Phys.* **07** (2001) 042 [hep-th/0104236].

[9] E.A. Bergshoeff, A. Bilal, M. de Roo and A. Sevrin, *Supersymmetric non-abelian Born-Infeld revisited*, *J. High Energy Phys.* **07** (2001) 029 [hep-th/0105274].

[10] L. De Fossé, P. Koerber and A. Sevrin, *The uniqueness of the abelian Born-Infeld action*, *Nucl. Phys.* **B 603** (2001) 413 [hep-th/0103015].

[11] J.M. Drummond, P.S. Howe and U. Lindström, *Kappa-symmetric non-abelian Born-Infeld actions in three dimensions*, hep-th/0206148.

[12] P. Koerber, to appear.

[13] P. Koerber and A. Sevrin, *The non-abelian Born-Infeld action through order* α'^3, *J. High Energy Phys.* **10** (2001) 003 [hep-th/0108169].

[14] A. Collinucci, M. De Roo and M.G.C. Eenink, *Supersymmetric Yang-Mills theory at order* α'^3, *J. High Energy Phys.* **06** (2002) 024 [hep-th/0205150].

[15] P. Koerber and A. Sevrin, *Testing the* $O(\alpha'^3)$ *term in the non-abelian open superstring effective action*, *J. High Energy Phys.* **09** (2001) 009 [hep-th/0109030];
M. de Roo, M.G.C. Eenink, P. Koerber and A. Sevrin, *Testing the fermionic terms in the non-abelian D-brane effective action through order* α'^3, *J. High Energy Phys.* **08** (2002) 011 [hep-th/0207015].

[16] K. Uhlenbeck and S.-T. Yau, *On the existence of hermitian Yang-Mills connections on stable vectorbundles*, *Comm. Pure Appl. Math.* **39** (1986) 257;
K. Uhlenbeck and S.-T. Yau, *A note on our previous paper: on the existence of hermitian Yang-Mills connections on stable vectorbundles*, *Comm. Pure Appl. Math.* **42** (1989) 703;
S.K. Donaldson, *Infinite determinants, stable bundles and curvature*, *Duke Math. J.* **54** (1987) 231;
see also chapter 15 in the second volume M.B. Green, J.H. Schwarz and E. Witten, *Superstring theory*, Cambridge University Press, Cambridge 1986.

[17] E. Corrigan, C. Devchand, D.B. Fairlie and J. Nuyts, *First order equations for gauge fields in spaces of dimension greater than four*, *Nucl. Phys.* **B 214** (1983) 452.

[18] A.A. Tseytlin, *On non-abelian generalisation of the Born-Infeld action in string theory*, *Nucl. Phys.* **B 501** (1997) 41 [hep-th/9701125].

[19] O.D. Andreev and A.A. Tseytlin, *Partition function representation for the open superstring effective action: cancellation of Möbius infinities and derivative corrections to Born-Infeld lagrangian*, *Nucl. Phys.* **B 311** (1988) 205.

[20] N. Wyllard, *Derivative corrections to D-brane actions with constant background fields*, *Nucl. Phys.* **B 598** (2001) 247 [hep-th/0008125].

[21] A. Hashimoto and I.W. Taylor, *Fluctuation spectra of tilted and intersecting D-branes from the Born-Infeld action*, *Nucl. Phys.* **B 503** (1997) 193 [hep-th/9703217].

[22] F. Denef, A. Sevrin and J. Troost, *Non-abelian Born-Infeld versus string theory*, *Nucl. Phys.* **B 581** (2000) 135 [hep-th/0002180].

[23] A. Sevrin, J. Troost and W. Troost, *The non-abelian Born-Infeld action at order F^6*, *Nucl. Phys.* **B 603** (2001) 389 [hep-th/0101192].

[24] S. Stieberger and T.R. Taylor, *Non-abelian Born-Infeld action and type-I–heterotic duality, 1. Heterotic F^6 terms at two loops*, hep-th/0207026.